Estimation of Term Structure of Defaultable Bonds Based on Kalman Filter Method

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Abstract: the Term Structure of Defaultable Bond Interest Rates Has Always Been a Research Hotspot in the Field of Financial Engineering, and the Model Estimates Involved Are the Basis and Key Links. with the Characteristics of Easy Operation, High Stability, Fast and Accurate, Kalman Filtering Method Has a Great Application Prospect in the Estimation of the Term Structure of Defaultable Bond Interest Rates. in View of This, This Paper Deeply Analyzes the Research Status of the Term Structure Model of the Defaultable Bond Interest Rate, Which Refers to the Emergence of the Stage Interest Rate Term Structure Model Mainly Divided into Two Categories: the Equilibrium Model and the No-Arbitrage Model, and Analyze Their Respective Characteristics. in This Context, the Application Theory of Kalman Filter Estimation Method is Taken as the Entry Point, and the Specific Application of Kalman Filter Method in the Estimation of the Term Structure of Defaultable Bond Interest Rate is Analyzed, and the Corresponding Maximum Likelihood Function is Obtained.

1. Introduction

1.1 Literature Review

As one of the important topics in the field of financial research, the term structure of interest rates has been a fundamental research work in the financial field. Its model estimation is the basic link for conducting theoretical and empirical research. Wu Jianhua, Zhang Ying and others constructed a Bayesian layered model of the term structure of defaultable bond interest rates, and used Dirichlet multi-layer prior distribution to obtain different credit yield curves. Several scholars further studied the exchange market and empirically tested the validity of the model. The research results show that the constructed model can overcome the influence of small samples and outliers and improve the forecasting performance. The Biy yield curve is interlaced and has certain reference value (Wu et al, 2017). Zhang Qing pointed out that the interest rate level should be determined by the market, and the profit term structure represents the level of supply and demand of funds to a certain extent. Zhang cleaned through the Svensson model, calculated different out-of-term spot interest rates, and applied it to the single-factor and two-factor CIR models to estimate, the results show that the two-factor model is more effective (Zhang, 2018). Wang Xuebiao, Zhang Qisong believes that dynamic estimation and static estimation are the basic methods for estimating the term structure of interest rates, and the latter is the basis for the former analysis. Two scholars proposed an improved genetic algorithm by analyzing the cubic spline function. The results show that the new algorithm has stronger predictive ability (Wang and Zhang, 2018). Wang Feiting used the Nelson-Siegel model to study the bond coupons of the above-mentioned exchanges, estimated the term structure of interest rates, and used principal component analysis to extract the influencing factors of the term structure of interest rates (Wang, 2017). Shen Yue pointed out that the study of the term structure of different bond interest rates needs to model the homeopathic interest rate and objectively analyze the operations such as risk hedging (Shen, 2017). Zhang Hanfei and Liu Haidong pointed out that the current interest rate level in China belongs to the official interest rate, and the term structure of the interest rate is estimated in advance according to the price of the transaction bond on the market (Zhang and Liu, 2008).

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1.2 Purpose of the Study

The term structure of interest rates can reflect the impact of time on interest rates to a certain extent. Therefore, the term structure can be used as the benchmark for the financial system. Since the expected return of risky assets can also be expressed by the excess return of risk-free interest rates, the term structure of interest rates can also be used as the basis for asset pricing. Due to the important position of the term structure of interest rates in the financial field, the theoretical and structural models related to it are the focus of research. As a key link, many scholars have conducted research around the estimation of the profit term structure model. At this stage, the maximum likelihood method and the generalized distance estimation method are most supported. Then, how to construct the maximum likelihood estimation function and obtain the model parameter estimation value is worth further research, and the Kalman class filtering method can effectively achieve this goal. Analysis of existing literature at home and abroad shows that there is very little research. For this reason, this paper explores relevant issues.

2. Default Bond Model of Defaultable Bond Interest Rate

At present, there are many research models around the term structure of interest rates, which can be divided into two categories in general. One is the equilibrium model, which is based on the theory of liquidity preference (Vasicek and Niu, 2018). The other type is the no-arbitrage model, which is derived from its theory.

In the equilibrium model, the specific process of the dynamics of the economic variables is assumed, and the change of the term structure under the assumptions is given. According to the number of impact factors, the equilibrium model can be divided into single factor and multi-factor equilibrium models. In order to facilitate the discussion of the article, the mathematical symbols commonly used in the model are first described, as shown in Table 1.

Table 1 Model Common Mathematical Symbols

Symbol	Meaning
r(t)	Instant interest rate at time t
P(t,T)	At time t, the price of a defaultable bond with a maturity of T
R(t,T)	Under the continuous compound interest calculation, the yield of the defaultable bond with a maturity of T at t
f(t,T)	Instantaneous forward rate of a defaultable bond with a maturity of T at time t
μ(r)	Short-term change in interest rate drift
σ(r)	Instantaneous volatility

The short-term profit calculation formula is:

$$dr_1 = \mu(r_1)dt + \sigma(r_1)W_t \qquad (1)$$

Observing this formula, we can see that the Brownian motion change can be split in the short term, one part is the drift amount $\mu(r_1)dt$ from t to t+dt, and the other part is the random impact $\sigma(r_1)W_t$ represented by Brownian motion, both of which are only related to the current interest rate level.

This means that the interest rate has an infinite term, that is, if the current interest rate level contains all the past interest rate information, the past information cannot help predict the future interest rate, and the interest rate change completely follows the random walk principle.

The process of deriving the equilibrium model determines that it does not have an arbitrage opportunity, otherwise the market cannot reach equilibrium. Although the equilibrium model parameters can be obtained from historical data regression, since the behavioral pattern of the default bond interest rate structure is in the process of dynamic change, it is difficult to predict the current market price by relying on the simulation of historical data, which presents an arbitrage opportunity. Therefore, many scholars have constructed a large number of no-arbitrage models, and the larger ones are Ho-Lee model, Hull-White model, and Heath-Jarrow-Morton model. Among the three models, the Ho-Lee model advocates that under the assumption of no-arbitrage opportunities, the current term structure of interest rates already contains enough information to predict its specific changes. This model contains two aspects, one is the structure of the initial interest rate term, and the other is the arbitrage constraint with interest rate changes. The Hull-White model has a mean-wall recovery feature, and its future volatility of short-term interest rates is random. Moreover, since the short-term interest rate shows a normal distribution law, the short-term interest rate will have a negative number in the form of a positive probability. The Heath-Jarrow-Morton model differs in that it uses the characteristics of intertemporal fluctuations as the entry point to set the volatility function structure of the relevant bond. Since the specific form of the model depends on the form of the wave, the model is rich. And, to a certain extent, the model can be regarded as a promotion of the Ho-Lee model, and it also has the disadvantage of possibly generating negative interest rates and infinite interest rates.

3. Application Theory of Kalman Class Filter Estimation Method

The Kalman filter method was first proposed by Kalman et al. in 1961. By introducing state variables and space concepts, a new state space method in the time domain was formally born. The Kalman filter method is suitable for processing multivariable systems and has great advantages in the processing of signal estimation problems. The specific operation process is to first describe the dynamic system by using the state equation, and then describe the state observation information by observing the equation. On this basis, the state is regarded as “point”, and the Hibert theory is used to estimate the optimal state.

In the Kalman filter estimation, the estimated system satisfies the following linear stochastic differential equation, which is essentially a discrete-time system that provides state estimation for stochastic systems including noise and noise measurements and autoregressive analysis.

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{2}$$

$$z_k = Hx_k + v_k \tag{3}$$

In the formula, x is the state vector, w is the process noise vector, z is the observation vector, and the observation noise vector is represented by v. The four variables are independent of each other, and $w - N(0, Q)$, $v - N(0, R)$.In practical application, the measurement noise covariance and noise covariance matrix are in a dynamic state. In order to facilitate qualitative research, this paper assumes it as a constant.

In the above system equation, A is the concrete state transition matrix, whose dimension is $n*n$, and the corresponding state vector when $k-1$ changes to k is the time state vector. B is the corresponding input control matrix, whose dimension is $m*n$, which represents the import process of the control system. H is the corresponding observation matrix, and its dimension is s. In practical application, the three are also in the state of changing at any time. In order to simplify the goal, they are also assumed to be constant.

The Kalman filtering method does not require past observation data, and has the advantages of simple calculation and no dependence on a large amount of data. But at the same time, it also has the following disadvantages: First, it requires a large amount of calculation and storage. Second, the accuracy of mathematical models and noise statistics is high. Third, Kalman filtering is a linear method with limited application range.

4. The Specific Application of Kalman Class Filter Estimation Method in the Estimation of the Term Structure of Default Bond Interest Rate

Whether it is a no-arbitrage model or a balanced model, the one-factor model can be expressed as:

$$dr_t = (a_t + b_t r_t)dt + \sigma_t r_t^{\gamma} dW_t \tag{4}$$

In the formula, r_t is the interest rate, dW_t is the Brownian motion, and the others are pending parameters.

When $a_t = \theta(t)$, $b_t = -a$, $\sigma_t = \sigma$, $\gamma = 0$,which is Hull-White mode.

$$dr(t) = \left[\theta(t) + a(t)(b - r(t))\right]dt + \sigma(t)\sqrt{r(t)}dW_t \quad (5)$$

Because dW_t is a standard Brownian motion, there is $dW_t = \varepsilon\sqrt{dt}$. In the formula, $\varepsilon - N(0,1)$.

Dividing both sides by V simultaneously yields:

$$r_t = b_t r_t + a_t + \frac{\sigma_t r_t^{\gamma}}{\sqrt{\Delta t}}\varepsilon \quad (6)$$

At present, China announces that the interest rate level is different from the market interest rate and sets the interest rate for the government. Let r_t be the current period and tw the profit rate. When the face value is 100, the price of the defaultable bond at the time limit t is:

$$P_t = 100ecp(-r_t t) \quad (7)$$

Where t is the expiration date and P is the specific price. Considering that the observation points are not continuous, equation (7) should be in discrete form, that is:

$$P_t = 100ecp(-r(t_k)t_k), k = 1,2,3,...J \quad (8)$$

In the formula, t_k is the specific period, and J is the number of observation points.

The observation error term e is introduced to reflect the degree of matching with the observation value. Using Kalman filter estimation, the calculation in the substitution formula, the maximum likelihood function is:

$$L_Z(\theta) = -\frac{dJ}{2}\ln(2\pi) - \frac{1}{2}\sum_{k=1}^{J}\ln\left|\sum k\right| - \frac{1}{2}\sum_{k=1}^{J}v_k{}'\sum k^{-1}v_k \quad (9)$$

Where d is the observation dimension, θ is the parameter vector to be estimated, and J is the number of observation points.

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